BAULKHAM HILLS HIGH SCHOOL

2022 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5 – 11)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 What is the length of the vector $-2\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k}$?

A. 1

B. 7

C. 11

D. 49

2 What is the converse of the following statement?

"*If a point is in the first quadrant, then its coordinates are positive*".

A. If a point is not in the first quadrant, then its coordinates are positive.

B. If a point is not in the first quadrant, then its coordinates are not positive.

C. If the coordinates of a point are positive, then the point is in the first quadrant.

D. If the coordinates of a point are not positive, then the point is not in the first quadrant.

3 Multiplying a non-zero complex number by $\frac{1+i}{1-i}$ results in a rotation about the origin on an Argand diagram. What is the rotation?

A. Clockwise by $\frac{\pi}{4}$ radians.

B. Clockwise by $\frac{\pi}{2}$ radians.

C. Anticlockwise by $\frac{\pi}{4}$ radians.

D. Anticlockwise by $\frac{\pi}{2}$ radians.

4 Which expression is equal to $\int \cot x \, dx$?

A. $\ln|\sin x| + c$

B. $\dfrac{\cot^2 x}{2} + c$

C. $\text{cosec}^2 x + c$

D. $\ln|\text{cosec}\, x| + \cot x + c$

5 What is the angle, to the nearest degrees, between the vectors $\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$ and $-2\underset{\sim}{i} - 4\underset{\sim}{j} + 6\underset{\sim}{k}$?

A. 0

B. 119

C. 136

D. 180

6 Which expression is equal to $\displaystyle\int \frac{1}{\sqrt{-x^2 + 6x - 5}} dx$?

A. $\sin^{-1}\left(\dfrac{x-3}{2}\right) + C$

B. $\cos^{-1}\left(\dfrac{x-3}{2}\right) + C$

C. $\ln\left|x - 3 + \sqrt{(x-3)^2 + 4)}\right| + C$

D. $\ln\left|x - 3 + \sqrt{(x-3)^2 - 4)}\right| + C$

7 Consider the statement:

"*If my computer is not working, then I cannot finish my homework*".

What is the negation of the above statement?

A. My computer is working, and I can finish my homework.

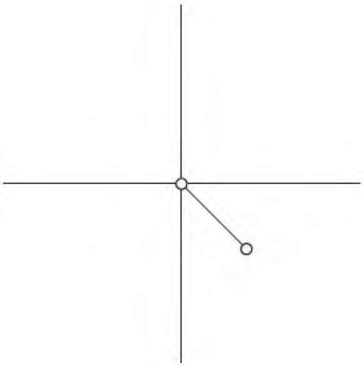
B. My computer is working, but I cannot finish my homework.

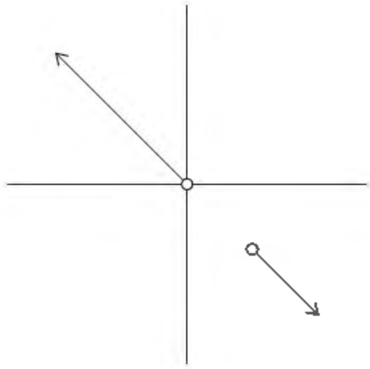
C. My computer is not working, and I can finish my homework.

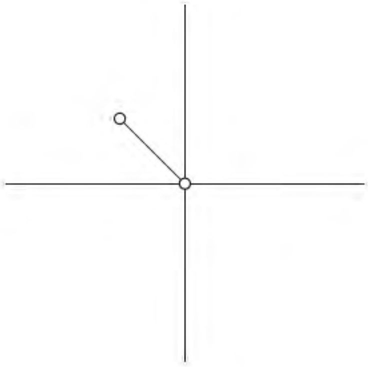
D. My computer is not working, but I cannot finish my homework.

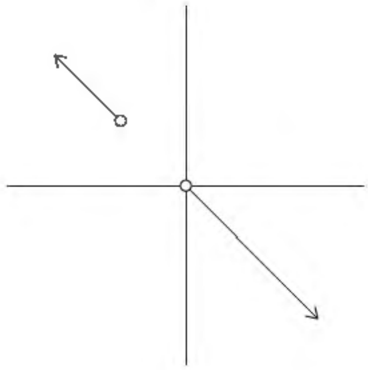
8 Which diagram best represents the solutions to the equation

$$\arg\left(\frac{z}{z-1+i}\right) = \pi \ ?$$

A. 

B. 

C. 

D. 

9 A particle is moving in simple harmonic motion with period 3 and amplitude 2. Which is a possible expression for the velocity, v, of the particle?

A. $v = \frac{4\pi}{3}\cos\left(\frac{2\pi}{3}t\right)$

B. $v = 2\cos\left(\frac{2\pi}{3}t\right)$

C. $v = \frac{2\pi}{3}\cos\left(\frac{\pi}{3}t\right)$

D. $v = 2\cos\left(\frac{\pi}{3}t\right)$

10 Which of the following statements is **false**?

A. $\forall x \in \mathbb{R}, x^2 \geq 0$

B. $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}, m = n + 5$

C. $\exists a \in \mathbb{R}, \forall x \in \mathbb{R}, ax = x$

D. $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, m = n + 5$

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question on the appropriate pages of the answer booklet. Extra paper is available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks) Use the pages labelled Question 11 in the answer booklet.

(a) Consider the complex numbers $w = -2 + 3i$ and $z = 1 + i$.

(i) Evaluate $\frac{1}{|w|}$. **1**

(ii) Express $w\bar{z}$ in the form $a + bi$, where a and b are real numbers. **2**

(b) (i) Express $z = \sqrt{2} - \sqrt{2}i$ in modulus-argument form. **2**

(ii) Hence find z^{21} in the form $a + bi$, where a and b are real numbers. **2**

(c) Use integration by parts to find $\int x^3 \log_e x \, dx$. **2**

(d) (i) Find the real numbers a and b, such that $\frac{5x^2 - 3x + 13}{(x-1)(x^2+4)} \equiv \frac{a}{x-1} + \frac{bx-1}{x^2+4}$. **2**

(ii) Hence, find $\int \frac{5x^2 - 3x + 13}{(x-1)(x^2+4)} dx$. **2**

(e) Using the substitution $t = \tan\frac{x}{2}$, or otherwise, calculate **3**

$$\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 3\sin x + 4\cos x}.$$

End of Question 11

Question 12 (16 marks) Use the pages labelled Question 12 in the answer booklet.

(a) Consider the vectors $\underset{\sim}{a} = \begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix}$ and $\underset{\sim}{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$.

(i) Find $\underset{\sim}{a} \cdot \underset{\sim}{b}$. **1**

(ii) Find the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$. **2**

(b) Consider two points $P(-1, 3, 1)$ and $Q(2, 4, 5)$ in three-dimensional space.

(i) Find the midpoint of PQ. **1**

(ii) If P and Q are the endpoints of the diameter of a sphere, find a vector equation of the sphere. **2**

(c) Find the Cartesian equation of the line $\underset{\sim}{r} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + \lambda(2\underset{\sim}{i} + \underset{\sim}{j})$. **2**

(d) Consider the following proposition:

"Suppose $x, y \in \mathbb{R}$. If $y^3 + yx^2 \le x^3 + xy^2$, then $y \le x$."

(i) State the contrapositive of this proposition. **1**

(ii) Hence prove the above proposition is true. **2**

Question 12 continues on page 7

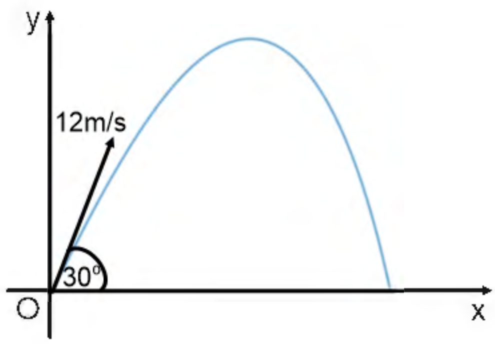
(e) A projectile is fired from the origin O with initial velocity 12 ms^{-1} at an angle of 30° to the horizontal and experiences air resistance proportional to the velocity in both the x and y directions. The equations of motion are given by

$$\ddot{x} = -k\dot{x} \qquad \ddot{y} = -10 - k\dot{y}$$

$$\dot{x} = 6\sqrt{3}e^{-kt} \qquad \dot{y} = \frac{1}{k}[(10+6k)e^{-kt} - 10]$$

$$x = \frac{6\sqrt{3}}{k}(1-e^{-kt}) \qquad y = \frac{10+6k}{k^2}(1-e^{-kt}) - \frac{10}{k}t$$

where k is a positive constant. (Do NOT prove these results.)



(i) Show that the Cartesian equation of the flight path is given by **2**

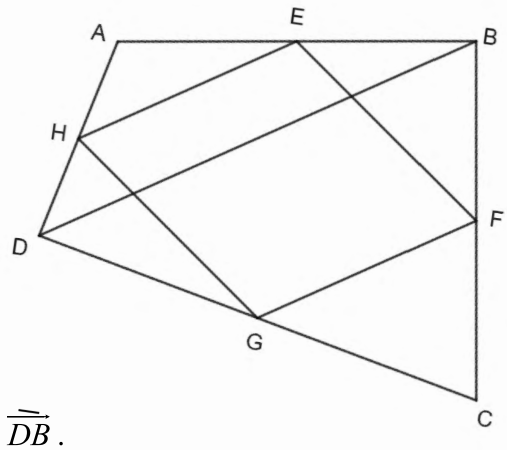
$$y = \frac{5+3k}{3k\sqrt{3}}x + \frac{10}{k^2}\ln\left(1 - \frac{kx}{6\sqrt{3}}\right)$$

(ii) Given that $k = 0.1$, find the maximum height reached in metres to 2 decimal places. **3**

End of Question 12

Question 13 (15 marks) Use the pages labelled Question 13 in the answer booklet.

(a) Varignon's theorem states that the midpoints of any quadrilateral join to form a parallelogram. Let $ABCD$ be a quadrilateral and $EFGH$ be the quadrilateral obtained by joining the midpoints of the edges of $ABCD$.



(i) Show that $\overrightarrow{HE} = \frac{1}{2}\overrightarrow{DB}$. **2**

(ii) Hence prove Varignon's theorem, i.e. show that $EFGH$ is a parallelogram. **1**

(b) Let $1, \omega, \omega^2$ be the cube roots of 1 and $\omega \neq 1$.

(i) State the values of ω^3 and $1 + \omega + \omega^2$. **1**

(ii) Hence evaluate $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)(1-\omega^7)(1-\omega^8)$. **2**

(c) (i) Graph the locus of z if $|z - 2i| = 1$. **2**

(ii) Hence, or otherwise, find the maximum and minimum values of the argument of z. **2**

(d) Consider the geometric series

$$S_{2n} = 1 - h + h^2 - \cdots + h^{2n}, \text{ where } 0 < h < 1.$$

(i) Show that $S_{2n-1} < \frac{1}{1+h} < S_{2n}$. **2**

(ii) Integrate the previous result between $h = 0$ and $h = x$, where $0 < x < 1$, and hence write down a polynomial inequality for $\log_e(1 + x)$. **2**

(iii) Use $n = 3$ to estimate the value of $\log_e \frac{5}{4}$ to 4 significant figures. Show full working. **1**

End of Question 13

Question 14 (14 marks) Use the pages labelled Question 14 in the answer booklet.

(a) Let $P(z) = z^8 - \frac{5}{2}z^4 + 1$. The complex number w is a root of $P(z) = 0$.

(i) Show that iw and $\frac{1}{w}$ are also roots of $P(z) = 0$. **1**

(ii) Hence, or otherwise, find all roots of $P(z) = 0$. **2**

(b) Find a unit vector that is perpendicular to both $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$. **3**

(c) A ball of mass 2 kilograms is thrown vertically upward from the origin with an initial speed of 8 metres per second. The ball is subject to a downward gravitational force of 20 newtons and an air resistance of $\frac{v^2}{5}$ newtons in the opposite direction to the velocity, v metres per second.

Hence, until the ball reaches its highest point, the equation of motion is

$$\ddot{y} = -\frac{v^2}{10} - 10, \qquad \text{(Do NOT prove this.)}$$

where y metres is its height.

(i) Show that, while the ball is rising, $v^2 = 164e^{-\frac{y}{5}} - 100$. **2**

(ii) Hence find the maximum height reached. **1**

(iii) Find how long the ball takes to reach this maximum height. **2**

(iv) Write down the equation of motion for the downwards journey. **1**

(v) How fast is the ball travelling when it returns to the origin? **2**

End of Question 14

Question 15 (15 marks) Use the pages labelled Question 15 in the answer booklet.

(a) For $n = 0, 1, 2, \ldots$, let $I_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$.

(i) Show that $I_1 = \frac{1}{2}\ln 2$. **1**

(ii) Show that for $n \geq 2$, $I_n + I_{n-2} = \frac{1}{n-1}$. **2**

(iii) For $n \geq 2$, explain why $I_n < I_{n-2}$, and deduce that **3**

$$\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}.$$

(iv) By using the recurrence relation of part (ii), find I_5 and deduce that **2**

$$\frac{2}{3} < \ln 2 < \frac{3}{4}.$$

(b) A particle moves in a straight line. Its displacement, x metres, after t seconds is given by

$$x = \sqrt{2}\sin\left(3t - \frac{\pi}{4}\right) + 3\sin(3t)$$

(i) Prove that the particle is moving in simple harmonic motion. **2**

(ii) Find the amplitude and period of the motion. **3**

(iii) When is the first time the particle reaches the origin? **2**

End of Question 15

Question 16 (14 marks) Use the pages labelled Question 16 in the answer booklet.

(a) Find $\int \frac{\sqrt{25x^2 - 4}}{x} dx$. **3**

(b) Let $\underset{\sim}{m}$ and $\underset{\sim}{n}$ be two vectors such that $\underset{\sim}{m} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$ and $\underset{\sim}{n} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$. **2**

If $a + b + c = 9$, prove $a^2 + b^2 + c^2 \geq 27$, where $a, b, c \in \mathbb{R}$.

(c) If $a, b, c > 0$, prove that $\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} \geq \frac{3}{2}$. **3**

(d) (i) If $y = x^k + (c - x)^k$, where $c > 0$, $k > 0$, $k \neq 1$, show that y has a single stationary point between $x = 0$ and $x = c$. **2**

(ii) Show that the stationary point is a maximum if $k < 1$ and a minimum if $k > 1$. **1**

(iii) Hence show that if $a > 0$, $b > 0$, $a \neq b$, then; **3**

$\frac{a^k + b^k}{2} < \left(\frac{a+b}{2}\right)^k$ if $0 < k < 1$ and $\frac{a^k + b^k}{2} > \left(\frac{a+b}{2}\right)^k$ if $k > 1$.

End of paper

BAULKHAM HILLS HIGH SCHOOL

2022 YEAR 12 EXTENSION 2 TRIAL HSC SOLUTIONS

	Solution	Marks
Section I		**1**
1. B	$\sqrt{(-2)^2+(-3)^2+6^2}=7$	**1**
2. C	Converse of $P\to Q$ is $Q\to P$	**1**
3. D	$\frac{1+i}{1-i}\times\frac{1+i}{1+i}$ $=\frac{1+2i+i^2}{1-i^2}$ $=\frac{2i}{2}$ $=i$ $\therefore$ An anti-clockwise rotation by $\frac{\pi}{2}$.	**1**
4. A	$\int\cot x\,dx$ $=\int\frac{\cos x}{\sin x}dx$ $=\ln\lvert\sin x\rvert+c$	**1**
5. D	$\cos\theta=\frac{\underset{\sim}{u}.\underset{\sim}{v}}{\lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert}$ $\cos\theta=\frac{1\times(-2)+2\times(-4)+(-3)\times6}{\sqrt{1^2+2^2+(-3)^2}\times\sqrt{(-2)^2+(-4)^2+6^2}}$ $=\frac{-28}{\sqrt{14}\times\sqrt{56}}$ $=-1$ $\theta=180^o$	**1**
6. A	$-x^2+6x-5\quad=-(x^2-6x+5)$ $=-[(x-3)^2-4]$ $=4-(x-3)^2$ $\therefore\int\frac{1}{\sqrt{-x^2+6x-5}}dx$ $=\int\frac{1}{\sqrt{2^2-(x-3)^2}}dx$ $=\sin^{-1}\left(\frac{x-3}{2}\right)+C$	**1**

7. C	$\neg(P \to Q)$ $= \neg(\neg P \vee Q)$ $= \neg\neg P \wedge \neg Q$ $= P \wedge \neg Q$	**1**
8. A	$\arg\left(\frac{z}{z-1+i}\right) = \pi$ $\arg(z) - \arg(z-(1-i)) = \pi$ ∴ The difference in angle is π. ∴ z lies between (0,0) and (1,-1).	**1**
9. A	Consider answer A: $v = \frac{4\pi}{3}\cos\left(\frac{2\pi}{3}t\right)$ $x = 2\pi\sin\left(\frac{2\pi}{3}t\right) + C$ ∴Amplitude is 2 and Period is $\frac{2\pi}{\left(\frac{2\pi}{3}\right)} = 3$. (B, C and D does not work).	**1**
10. D	A) For every real number x, $x^2 \geq 0$. True B) For all integers *n*, there exist an integer *m* such that $m = n + 5$. True C)There exists a real number *a* for which *ax*=*x* for every real number *x*. True D) There exists an integer *m*, for all integers *n* such that $m = n + 5$. False	**1**

Question	Solution	Mark	Comments
Section II			
11(a)(i)	$\frac{1}{\lvert w\rvert} = \frac{1}{\sqrt{(-2)^2+3^2}} = \frac{1}{\sqrt{13}}$	**1**	1Mk: Provides correct solution.
11(a)(ii)	$w\bar{z} = (-2+3i)(1-i)$ $= -2 + 2i + 3i - 3i^2$ $= 1 + 5i$	**2**	2Mk: Provides correct solution. 1Mk: Obtains correct $\bar{z}$.
11(b)(i)	$z = \sqrt{2} - \sqrt{2}i$ $\lvert z\rvert = \sqrt{2+2} = 2$ & $\arg(z) = \tan^{-1}\left(\frac{-\sqrt{2}}{\sqrt{2}}\right) = -\frac{\pi}{4}$ $\therefore z = 2\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$	**2**	2Mk: Provides correct solution. 1Mk: Obtains correct modulus or argument.

11(b)(ii)	$z^{21}=2^{21}\left(\cos\left(21\times-\frac{\pi}{4}\right)+i\sin\left(21\times-\frac{\pi}{4}\right)\right)$ $(De\ Moivre's\ Theorem)$ $=2^{21}\left(\cos\left(-\frac{21\pi}{4}\right)+i\sin\left(-\frac{21\pi}{4}\right)\right)$ $=2^{21}\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)$ $=2097152\left(-\frac{1}{\sqrt{2}}+i\left(-\frac{1}{\sqrt{2}}\right)\right)$ $=-1048576\sqrt{2}+1048576\sqrt{2}i$	**2**	2Mk: Provides correct solution. 1Mk: Uses De Moivre's Theorem correctly.
11(c)	$\int x^3 \ln x\,dx$ $u=\ln x \quad v=\frac{1}{4}x^4$ $du=\frac{dx}{x} \quad dv=x^3$ $=\left(\frac{1}{4}x^4\right)(\ln x)-\int\frac{1}{4}x^3\,dx$ $=\frac{1}{4}x^4\ln x-\frac{x^4}{16}+C$	**2**	2Mk: Provides correct solution. 1Mk: Attempts to use integration by parts.
11(d)(i)	$\frac{5x^2-3x+13}{(x-1)(x^2+4)}\equiv\frac{a}{x-1}+\frac{bx-1}{x^2+4}$ $=\frac{a}{x-1}+\frac{bx-1}{x^2+4}$ $=\frac{a(x^2+4)+(bx-1)(x-1)}{(x-1)(x^2+4)}$ $\therefore a(x^2+4)+(bx-1)(x-1)=5x^2-3x+13$ When $x=1,\ 5a=15$ $a=3$ Equating the coefficient of x^2 gives $a+b=5$ $b=2$ $\therefore a=3, b=2$	**2**	2Mk: Provides correct solution. 1Mk: Obtains *a* or *b* correctly.
11(d)(ii)	$\int\frac{5x^2-3x+13}{(x-1)(x^2+4)}\,dx$ $=\int\frac{3}{x-1}+\frac{2x-1}{x^2+4}dx$ $=3\ln\|x-1\|+\int\frac{2x\,dx}{x^2+4}-\int\frac{dx}{x^2+4}$ $=3\ln\|x-1\|+\ln(x^2+4)-\frac{1}{2}\tan^{-1}\frac{x}{2}+C$	**2**	2Mk: Provides correct solution. 1Mk: Correctly integrates for a ln or inverse tan.

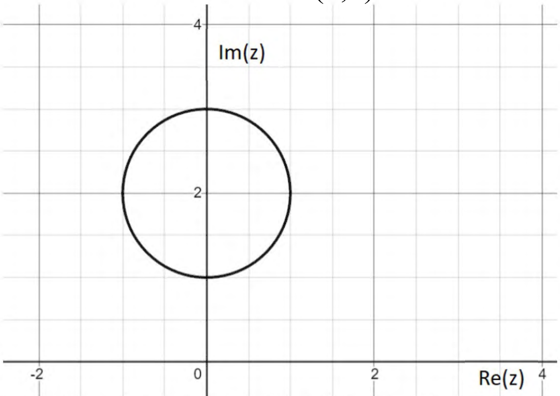
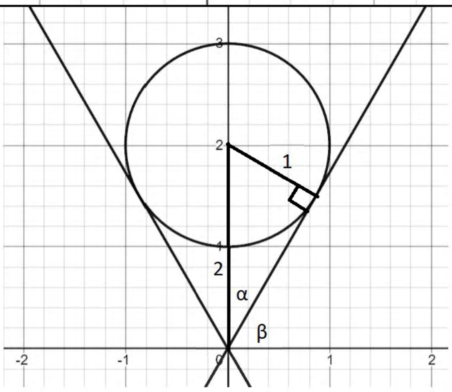
11(e)	$\int_0^{\frac{\pi}{2}} \frac{dx}{5+3\sin x+4\cos x}$ Let $t=\tan\frac{x}{2}$ $\therefore \frac{dt}{dx}=\frac{1}{2}\sec^2\frac{x}{2}$ $=\frac{1}{2}(\tan^2\frac{x}{2}+1)$ $\frac{dt}{dx}=\frac{1}{2}(t^2+1)$ $\therefore \frac{2dt}{1+t^2}=dx$ When $x=\frac{\pi}{2}$, $t=1$ When $x=0$, $t=0$ $\therefore = \int_0^1 \frac{2dt}{(1+t^2)\left[5+3\left(\frac{2t}{1+t^2}\right)+4\left(\frac{1-t^2}{1+t^2}\right)\right]}$ $=\int_0^1 \frac{2dt}{5+5t^2+6t+4-4t^2}$ $=\int_0^1 \frac{2dt}{(t+3)^2}$ $=-2\left[\frac{1}{t+3}\right]_0^1$ $=-2\left(\frac{1}{4}-\frac{1}{3}\right)$ $=\frac{1}{6}$	**3**	3Mk: Provides correct solution. 2Mk: Integrates correctly. 1Mk: Obtains correct integrand in *t*.

12(a)(i)	$\underset{\sim}{a}\cdot\underset{\sim}{b}=(-1)(3)+(-2)(-1)+(4)(2)$ $=-3+2+8$ $=7$	**1**	1Mk: Provides correct solution.
12(a)(ii)	$\lvert\underset{\sim}{b}\rvert=\sqrt{3^2+(-1)^2+2^2}$ $=\sqrt{9+1+4}$ $=\sqrt{14}$ $proj_{\underset{\sim}{b}}\underset{\sim}{a}=\dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{\lvert\underset{\sim}{b}\rvert\lvert\underset{\sim}{b}\rvert}\underset{\sim}{b}$ $=\dfrac{7}{\sqrt{14}\times\sqrt{14}}\underset{\sim}{b}$ $=\dfrac{1}{2}(3\underset{\sim}{i}-\underset{\sim}{j}+2\underset{\sim}{k})$ $=\dfrac{3}{2}\underset{\sim}{i}-\dfrac{1}{2}\underset{\sim}{j}+\underset{\sim}{k}$	**2**	2Mk: Provides correct solution. 1Mk: Obtains $\lvert\underset{\sim}{b}\rvert^2$ correctly.
12(b)(i)	Midpoint $=\left(\dfrac{-1+2}{2},\dfrac{3+4}{2},\dfrac{1+5}{2}\right)$ $=\left(\dfrac{1}{2},\dfrac{7}{2},3\right)$	**1**	1Mk: Provides correct solution.
12(b)(ii)	Radius is the distance from midpoint to P: $=\sqrt{\left(\dfrac{1}{2}+1\right)^2+\left(\dfrac{7}{2}-3\right)^2+(3-1)^2}$ $=\sqrt{\dfrac{13}{2}}$ Equation of the sphere: $\left\lvert\underset{\sim}{r}-\begin{bmatrix}0.5\\3.5\\3\end{bmatrix}\right\rvert=\sqrt{\dfrac{13}{2}}$ Must be a vector equation.	**2**	2Mk: Provides correct solution. 1Mk: Obtains correct radius.
12(c)	$\underset{\sim}{r}=3\underset{\sim}{i}-4\underset{\sim}{j}+\lambda(2\underset{\sim}{i}+\underset{\sim}{j})$ $\therefore x=3+2\lambda$ $y=-4+\lambda$ $y+4=\lambda$ $\therefore x=3+2(y+4)$ $\therefore x-2y-11=0$ or $y=\dfrac{x}{2}-\dfrac{11}{2}$	**2**	2Mk: Provides correct solution. 1Mk: Attempts to solve simultaneously or finds the slope of the line.

12(d)(i)	Contrapositive is: If $y > x$, then $y^3 + yx^2 > x^3 + xy^2$	**1**	1Mk: Provides correct solution.
12(d)(ii)	Proving the contrapositive is true. I.e. if $y > x$ then $y^3 + yx^2 > x^3 + xy^2$. $y > x$ $y(x^2+y^2) > x(x^2+y^2) \quad (\text{since } x^2+y^2>0)$ $y^3 + yx^2 > x^3 + xy^2$ as required. $\therefore$ The contrapositive is true. By contraposition $\therefore$ If $y \le x$ then $y^3 + yx^2 \le x^3 + xy^2$	**2**	2Mk: Provides correct solution. 1Mk: Multiply both sides by (x^2+y^2) or correct algebraic method towards the inequality.
12(e)(i)	$\frac{kx}{6\sqrt{3}} = 1 - e^{-kt}$ $e^{-kt} = 1 - \frac{kx}{6\sqrt{3}}$ $-kt = \ln\left(1 - \frac{kx}{6\sqrt{3}}\right)$ $t = -\frac{1}{k}\ln\left(1 - \frac{kx}{6\sqrt{3}}\right)$ Substitute t into y: $y = \frac{10+6k}{k^2}.\frac{kx}{6\sqrt{3}} - \frac{10}{k}\left[-\frac{1}{k}\ln\left(1-\frac{kx}{6\sqrt{3}}\right)\right]$ $= \frac{10+6k}{6k^2\sqrt{3}}kx + \frac{10}{k^2}\ln\left(1-\frac{kx}{6\sqrt{3}}\right)$ $= \frac{5+3k}{3k\sqrt{3}}x + \frac{10}{k^2}\ln\left(1-\frac{kx}{6\sqrt{3}}\right)$ as required.	**2**	2Mk: Provides correct proof. 1Mk: Obtains correct t.

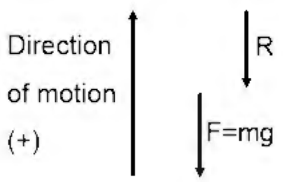
12(e)(ii)	Maximum height occurs when $\dot{y}=0$: $0=\frac{1}{k}[(10+6k)e^{-kt}-10]$ $(10+6k)e^{-kt}=10$ $e^{-kt}=\frac{10}{10+6k}$ $e^{-kt}=1+0.6k$ $kt=\ln(1+0.6k)$ $t=\frac{1}{k}\ln(1+0.6k)$ When $k=0.1$, $t=\frac{1}{0.1}\ln(1+0.6\times 0.1)$ $t=0.582689...$ Sub t into y: $y=\frac{10+6k}{k^2}(1-e^{-kt})-\frac{10}{k}t$ $y=\frac{10+6(0.1)}{0.1^2}(1-e^{-(0.1)(0.582689...)})-\frac{10}{0.1}(0.582689...)$ $y=1.73109...$ $y=1.73\text{m}$	**3**	3Mk: Provides correct solution. 2Mk: Obtains correct value for t or correct expression for t. 1Mk: Attempts to find t when $\dot{y}=0$.

13(a)(i)	$\overrightarrow{HE}=\overrightarrow{HA}+\overrightarrow{AE}$ $=\frac{1}{2}(\overrightarrow{DA}+\overrightarrow{AB})$ $=\frac{1}{2}(\overrightarrow{DB})$ as required.	**2**	2Mk: Provides correct solution. 1Mk: Correctly applies one addition rule.
13(a)(ii)	Similarly $\overrightarrow{GF}=\frac{1}{2}(\overrightarrow{DB})$ $=\overrightarrow{HE}$ $\therefore EFGH$ is a parallelogram. (One pair of opposite sides of a quadrilateral parallel and equal in length).	**1**	1Mk: Provides correct explanation.

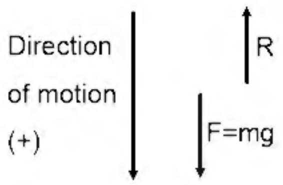
13(b)(i)	ω is the cube root of 1. $\therefore \omega^3 = 1$ $\omega^3 - 1 = 0$ $(\omega - 1)(\omega^2 + w + 1) = 0$ $\therefore \omega^2 + w + 1 = 0$	**1**	1Mk: Provides correct answers.
13(b)(ii)	$(1-\omega)(1-\omega^2)(1-w^4)(1-w^5)(1-w^7)(1-w^8)$ $=(1-\omega)(1-\omega^2)(1-w^1)(1-w^2)(1-w^1)(1-w^2)$ $=(1-w)^3(1-w^2)^3$ $=((1-w)(1-w^2))^3$ $=(1-w^2-w+w^3)^3$ $=(1+1+w^3)^3 \quad [-w^2-w=1 \text{ from (i)}]$ $=(1+1+1)^3 \quad [w^3=1 \text{ from (i)}]$ $=3^3$ $=27$	**2**	2Mk: Provides correct solution. 1Mk: Substitutes $w^3 = 1$.
13(c)(i)	It is a circle with centre (0,2) and radius 1.  	**2**	2Mk: Provides correct sketch. 1Mk: Gives correct shape.
13(c)(ii)	 For the minimum value of arg(z), it must be a tangent to the circle. Use trigonometry to solve for α and β. We get $\alpha = \frac{\pi}{6}$ and $\beta = \frac{\pi}{3}$. $\therefore$ Minimum $\arg(z) = \frac{\pi}{3}$. Similarly, maximum arg(z)= $\frac{2\pi}{3}$.	**2**	2Mk: Provides correct solution. 1Mk: Identify the minimum arg(z) is a tangent and finds the angle.

13(d)(i)	$S_{2n-1} = 1 - h + h^2 - h^3 + \ldots - h^{2n-1}$ $= \dfrac{1-(-h)^{2n}}{1-(-h)}$ (sum of GP) $= \dfrac{1-h^{2n}}{1+h}$ $< \dfrac{1}{1+h}$ (greater numerator) & $S_{2n} = 1 - h + h^2 - h^3 + \ldots + h^{2n}$ $= \dfrac{1-(-h)^{2n+1}}{1-(-h)}$ (sum of GP) $= \dfrac{1+h^{2n+1}}{1+h}$ $> \dfrac{1}{1+h}$ (smaller numerator) $\therefore\ S_{2n-1} < \dfrac{1}{1+h} < S_{2n}$ as required.	**2**	2Mk: Provides correct solution. 1Mk: Correctly proves one inequality.
13(d)(ii)	$\int_0^x S_{2n-1}\,dh < \int_0^x \dfrac{1}{1+h}\,dh < \int_0^x S_{2n}\,dh$ $\left[h - \dfrac{h^2}{2} + \ldots - \dfrac{h^{2n}}{2n}\right]_0^x < \left[\ln(1+h)\right]_0^x < \left[h - \dfrac{h^2}{2} + \ldots + \dfrac{h^{2n+1}}{2n+1}\right]_0^x$ $x - \dfrac{x^2}{2} + \ldots - \dfrac{x^{2n}}{2n} < \log_e(1+x) < x - \dfrac{x^2}{2} + \ldots + \dfrac{x^{2n+1}}{2n+1}$	**2**	2Mk: Provides correct solution. 1Mk: Integrates one side correctly, including obtaining $\log_e(1+x)$.
13(d)(iii)	Use $n = 3$ and $x = 0.25$ $0.25 - \dfrac{(0.25)^2}{2} + \dfrac{(0.25)^3}{3} - \dfrac{(0.25)^4}{4} + \dfrac{(0.25)^5}{5} - \dfrac{(0.25)^6}{6} < \ln(1+0.25) <$ $0.25 - \dfrac{(0.25)^2}{2} + \dfrac{(0.25)^3}{3} - \dfrac{(0.25)^4}{4} + \dfrac{(0.25)^5}{5} - \dfrac{(0.25)^6}{6} + \dfrac{(0.25)^7}{7}$ $0.223136\ldots < \ln\left(\dfrac{5}{4}\right) < 0.223145\ldots$ $\therefore \ln\left(\dfrac{5}{4}\right) = 0.2231$ (to 4 s.f.)	**1**	1Mk: Provides correct solution.

14(a)(i)	$P(z) = z^8 - \frac{5}{2}z^4 + 1$ and w is a root of $P(z) = 0$. Show that iw and $\frac{1}{w}$ are also roots of $P(z) = 0$. $P(iw) = (iw)^8 - \frac{5}{2}(iw)^4 + 1$ $= i^8 w^8 - \frac{5}{2} i^4 w^4 + 1$ $= w^8 - \frac{5}{2} w^4 + 1$ $= 0$ $P\left(\frac{1}{w}\right) = \left(\frac{1}{w}\right)^8 - \frac{5}{2}\left(\frac{1}{w}\right)^4 + 1$ $= \frac{1}{w^8}(1 - \frac{5}{2}w^4 + w^8)$ $= \frac{1}{w^8} \times 0$ $= 0$	**1**	1Mk: Correctly shows the results.
14(a)(ii)	$z^8 - \frac{5}{2}z^4 + 1 = 0$ $\therefore 2z^8 - 5z^4 + 2 = 0$ $\therefore (2z^4 - 1)(z^4 - 2) = 0$ $\therefore z^4 = \frac{1}{2} \text{ or } 2$ $\therefore$ One root is $z = \sqrt[4]{2}$ $\therefore\ z = \pm\sqrt[4]{2},\ z = \pm\sqrt[4]{2}\,i,\ z = \pm\frac{1}{\sqrt[4]{2}},\ z = \pm\frac{1}{\sqrt[4]{2}\,i}$	**2**	2Mk: Provides correct solution. 1Mk: Obtains one correct value for z.

14(b)	Let $\begin{bmatrix}x\\y\\z\end{bmatrix}$ be perpendicular to $\begin{bmatrix}3\\-2\\1\end{bmatrix}$ and $\begin{bmatrix}1\\4\\-1\end{bmatrix}$ $\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}.\begin{bmatrix}3\\-2\\1\end{bmatrix}=0 \quad \therefore 3x-2y+z=0$ $\therefore \begin{bmatrix}x\\y\\z\end{bmatrix}.\begin{bmatrix}1\\4\\-1\end{bmatrix}=0 \quad \therefore x+4y-z=0$ Solve simultaneously: $4x+2y=0$ $y=-2x$ Sub y into second equation: $x-8x-z=0$ $z=-7x$ $\therefore \begin{bmatrix}x\\-2x\\-7x\end{bmatrix}$ is perpendicular to the two given vectors. Let $x=1, \begin{bmatrix}1\\-2\\-7\end{bmatrix}$ is a one particular vector that is perpendicular. For unit vector: $\hat{\underset{\sim}{u}}=\frac{1}{\sqrt{1+4+49}}\begin{bmatrix}1\\-2\\-7\end{bmatrix}$ $=\frac{1}{\sqrt{54}}\begin{bmatrix}1\\-2\\-7\end{bmatrix}$ $=\frac{1}{3\sqrt{6}}\begin{bmatrix}1\\-2\\-7\end{bmatrix}$	**3**	3Mk: Provides correct unit vector. 2Mk: Obtains a general vector that is perpendicular to the given vectors. 1Mk: Obtains correct Cartesian equations.
14(c)	 $m\ddot{y}=-(F+R)$ $2\ddot{y}=-20-\frac{v^2}{5}$ $\ddot{y}=-10-\frac{v^2}{10}$		

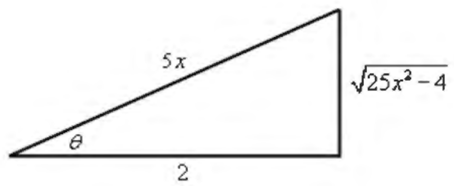
14(c)(i)	$\ddot{y} = v\dfrac{dv}{dy}$ $\therefore v\dfrac{dv}{dy} = -10 - \dfrac{v^2}{10}$ $\dfrac{dv}{dy} = \dfrac{-10}{v} - \dfrac{v}{10}$ $\dfrac{dy}{dv} = \dfrac{-10v}{v^2+100}$ $\therefore [y]_0^y = \int_8^v \dfrac{-10v}{v^2+100}dv$ $y = -5\left[\ln(v^2+100)\right]_8^v$ $-\dfrac{y}{5} = \ln(v^2+100) - \ln 164$ $-\dfrac{y}{5} = \ln\left(\dfrac{v^2+100}{164}\right)$ $\dfrac{v^2+100}{164} = e^{-\frac{y}{5}}$ $\therefore v^2 = 164e^{-\frac{y}{5}} - 100$ as required.	**2**	2Mk: Correctly shown. 1Mk: Attempts to integrate $\dfrac{-10v}{v^2+100}$.
14(c)(ii)	Max height occurs when $v = 0$. $\therefore 0 = 164e^{-\frac{y}{5}} - 100$ $100 = 164e^{-\frac{y}{5}}$ $-5\ln\left(\dfrac{100}{164}\right) = y$ $\therefore y = 5\ln(1.64)$ m or 2.47m	**1**	1Mk: Provides correct answer.
14(c)(iii)	$\ddot{y} = \dfrac{dv}{dt}$ $\therefore \dfrac{dv}{dt} = -10 - \dfrac{v^2}{10}$ $\dfrac{dt}{dv} = \dfrac{-10}{v^2+100}$ $\therefore t = \int_8^0 \dfrac{-10}{v^2+100}dv$ $= -10 \times \dfrac{1}{10}\left[\tan^{-1}\dfrac{v}{10}\right]_8^0$ $= \tan^{-1}\left[\dfrac{v}{10}\right]_0^8$ $= \tan^{-1}\left(\dfrac{4}{5}\right)$ $\therefore$ Reaches max height after $\tan^{-1}\left(\dfrac{4}{5}\right)$ seconds.	**2**	2Mk: Provides correct solution. 1Mk: Correctly integrates $\dfrac{-10}{v^2+100}$.

14(c)(iv)	 $m\ddot{y} = F - R$ $2\ddot{y} = 20 - \frac{v^2}{5}$ $\therefore \ddot{y} = 10 - \frac{v^2}{10}$	**1**	1Mk: Provides correct solution.
14(c)(v)	$v\frac{dv}{dy} = 10 - \frac{v^2}{10}$ $\frac{dv}{dy} = \frac{10}{v} - \frac{v}{10}$ $\frac{dy}{dv} = \frac{10v}{100 - v^2}$ $\left[y\right]_0^{5\ln 1.64} = \int_0^v \frac{10v}{100 - v^2}\,dv$ $5\ln 1.64 = -5\left[\ln(100 - v^2)\right]_0^v$ $-\ln 1.64 = \ln(100 - v^2) - \ln 100$ $\ln(100 - v^2) = \ln\left(\frac{100}{1.64}\right)$ $v^2 = \frac{64}{1.64}$ $v = \frac{8}{\sqrt{1.64}}$ (since $v > 0$) $v = 6.25$ m/s $\therefore$ Ball is travelling at 6.25 m/s when it returns to the origin.	**2**	2Mk: Provides correct solution. 1Mk: Integrates correctly.
15(a)(i)	$I_n = \int_0^{\frac{\pi}{4}} \tan^n\theta\,d\theta$ $I_1 = \int_0^{\frac{\pi}{4}} \tan\theta\,d\theta$ $= [-\ln(\cos\theta)]_0^{\frac{\pi}{4}}$ $= \ln\left[\cos(0)\right] - \ln\left[\cos\left(\frac{\pi}{4}\right)\right]$ $= \ln 1 - \ln\left(\frac{1}{\sqrt{2}}\right)$ $= 0 - \ln\left(2^{-\frac{1}{2}}\right)$ $= \frac{1}{2}\ln 2$ as required.	**1**	1Mk: Provides correct proof.

15(a)(ii)	Show that for $n \geq 2$, $I_n + I_{n-2} = \frac{1}{n-1}$. $I_n = \int_0^{\frac{\pi}{4}} \tan^{n-2}\theta(\sec^2\theta - 1)\,d\theta$, where $n \geq 2$ $= \int_0^{\frac{\pi}{4}} \tan^{n-2}\theta\sec^2\theta - \tan^{n-2}\theta\,d\theta$ $= \int_0^{\frac{\pi}{4}} \tan^{n-2}\theta\sec^2\theta\,d\theta - I_{n-2}$ $\therefore\ I_n + I_{n-2} = \int_0^{\frac{\pi}{4}} \tan^{n-2}\theta\sec^2\theta\,d\theta$ $= \left[\frac{\tan^{n-1}\theta}{n-1}\right]_0^{\frac{\pi}{4}}$ $= \frac{\left(\tan^{n-1}\theta\right)^{n-1}}{n-1} - \frac{\left(\tan(0)\right)^{n-1}}{n-1}$ $= \frac{1}{n-1} - 0$ $= \frac{1}{n-1}$	**2**	2Mk: Provides correct proof. 1Mk: Obtains correct integral for $I_n + I_{n-2}$ or uses integration by parts correctly towards I_n or I_{n-2}.
15(a)(iii)	For $0 < \theta < \frac{\pi}{4}$, $0 < \tan\theta < 1$ (since $\tan\theta$ is monotonic increasing on the domain) $0 < \tan^2\theta < 1$ $0 < \tan^n\theta < \tan^{n-2}\theta$ (since $\tan^{n-2} > 0$) $\therefore \int_0^{\frac{\pi}{4}} \tan^n\theta\,d\theta < \int_0^{\frac{\pi}{4}} \tan^{n-2}\theta\,d\theta$ that is, $I_n < I_{n-2}$ $\therefore 2I_n < I_{n-2} + I_n$ (1) $2I_n < \frac{1}{n-1}$ [from (ii)] $I < \frac{1}{2(n-1)}$ Replacing n by $n+2$ in (1), we have $I_{n+2} < I_n$ $I_{n+2} + I_n < 2I_n$ $\frac{1}{n+2-1} < 2I_n$ [from (ii)] $\frac{1}{2(n+1)} < I_n$ Hence $\frac{1}{2(n+1)} < I_n < \frac{1}{2(n-1)}$.	**3**	3Mk: Provides correct proof. 2Mk: Correctly explains $I_n < I_{n-2}$ and correctly proves part of the compound inequality or proving the compound inequality. 1Mk: Sufficiently explains why $I_n < I_{n-2}$.

15(a)(iv)	$I_5=\frac{1}{4}-I_3$ [from (ii)] $=\frac{1}{4}-\left(\frac{1}{2}-I_1\right)$ [from (ii)] $=-\frac{1}{4}+\frac{1}{2}\ln 2$ [from (i)] $=\frac{2\ln 2-1}{4}$ From (iii), we have $\frac{1}{2\times 6}<I_5<\frac{1}{2\times 4}$ $\frac{1}{12}<\frac{2\ln 2-1}{4}<\frac{1}{8}$ $\frac{1}{3}<2\ln 2-1<\frac{1}{2}$ $\frac{4}{3}<2\ln 2<\frac{3}{2}$ $\frac{2}{3}<\ln 2<\frac{3}{4}$ as required.	**2**	2Mk: Provides correct proof. 1Mk: Finds I_5 by using the recurrence relation.
15(b)(i)	$x=\sqrt{2}\sin\left(3t-\frac{\pi}{4}\right)+3\sin(3t)$ $\dot{x}=3\sqrt{2}\cos\left(3t-\frac{\pi}{4}\right)+9\cos(3t)$ $\ddot{x}=-9\sqrt{2}\sin\left(3t-\frac{\pi}{4}\right)-27\sin(3t)$ $=-9\left(\sqrt{2}\sin\left(3t-\frac{\pi}{4}\right)+3\sin(3t)\right)$ $=-3^2x$ In the form $\ddot{x}=-n^2x$, where $n=3$ and centre is 0. The motion is simple harmonic.	**2**	2Mk: Provides correct solution. 1Mk: Attempts to find $\dot{x}$ and $\ddot{x}$.
15(b)(ii)	Period is $\frac{2\pi}{3}$. Amplitude: $x=\sqrt{2}\sin\left(3t-\frac{\pi}{4}\right)+3\sin(3t)$ $=\sqrt{2}\left(\sin(3t)\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}\right)\cos(3t)\right)+3\sin(3t)$ $=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin(3t)-\frac{1}{\sqrt{2}}\cos(3t)\right)+3\sin(3t)$ $=-\cos(3t)+(1+3)\sin(3t)$ $=4\sin(3t)-\cos(3t)$ This can be expressed in the form $A\sin(3t-\alpha)$. $A\sin(3t-\alpha)=A\sin(3t)(\cos\alpha)-A\sin(\alpha)\cos(3t)$ $\therefore A\cos\alpha=4$ $A\sin\alpha=1$ $\therefore A=\sqrt{4^2+1^2}$ $=\sqrt{17}$ $\therefore$ Amplitude is $\sqrt{17}$	**3**	3Mk: Provides correct period and amplitude. 2Mk: Obtains correct period and significant progress towards amplitude or correct amplitude. 1Mk: Obtains correct period or significant progress towards amplitude.

15(b)(iii)	From (ii): $A\cos\alpha = 4$ $A\sin\alpha = 1$ $\tan\alpha = \frac{1}{4}$ $\alpha = \tan^{-1}\left(\frac{1}{4}\right)$ $\therefore \sqrt{2}\sin\left(3t-\frac{\pi}{4}\right)+3\sin(3t)=\sqrt{17}\sin\left(3t-\tan^{-1}\left(\frac{1}{4}\right)\right)$ Reaches the origin when $x=0$. $\sin\left(3t-\tan^{-1}\left(\frac{1}{4}\right)\right)=0$ $3t-\tan^{-1}\left(\frac{1}{4}\right)=0,\pi,2\pi,\ldots$ $3t=0+\tan^{-1}\left(\frac{1}{4}\right),\pi+\tan^{-1}\left(\frac{1}{4}\right),\ldots$ $t=\frac{\tan^{-1}\left(\frac{1}{4}\right)}{3},\frac{\pi+\tan^{-1}\left(\frac{1}{4}\right)}{3},\ldots$ $t=0.081659\ldots,1.128857\ldots,\ldots$ Using radians mode. $\therefore t=0.0817$s is the first time the particle is at the origin.	**2**	2Mk: Provides correct solution. 1Mk: Obtains correct value for α or uses $x=0$ to find an expression for time.

16(a)	$x=\frac{2}{5}\sec\theta \Rightarrow dx=\frac{2}{5}\sec\theta\tan\theta\, d\theta$ $\int\frac{\sqrt{25x^2-4}}{x}dx=\int\frac{2\tan\theta}{\frac{2}{5}\sec\theta}\left(\frac{2}{5}\sec\theta\tan\theta d\theta\right)$ $=2\int\tan^2\theta$ $=2\int\sec^2\theta-1d\theta$ $=2(\tan\theta-\theta)+C$ Since $\sec\theta=\frac{5x}{2}=\frac{\text{hyp}}{\text{adj}}$  $\therefore \tan\theta=\frac{\sqrt{25x^2-4}}{2}$ & $\cos\theta=\frac{2}{5x} \Rightarrow \theta=\cos^{-1}\left(\frac{2}{5x}\right)$ $\therefore \int\frac{\sqrt{25x^2-4}}{x}dx=\sqrt{25x^2-4}-2\cos^{-1}\left(\frac{2}{5x}\right)+C$	**3**	3Mk: Provides correct solution. 2Mk: Integrates correctly to get $2(\tan\theta-\theta)+C$. 1Mk: Uses a valid substitution.

16(b)	$\underset{\sim}{m} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}$ & $\underset{\sim}{n} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ $\therefore \underset{\sim}{m} \cdot \underset{\sim}{n} \le \lvert\underset{\sim}{m}\rvert\lvert\underset{\sim}{n}\rvert$ (from $\underset{\sim}{m} \cdot \underset{\sim}{n} = \lvert\underset{\sim}{m}\rvert\lvert\underset{\sim}{n}\rvert\cos\theta$) $\therefore a+b+c \le \sqrt{a^2+b^2+c^2} \times \sqrt{3}$ $9 \le \sqrt{a^2+b^2+c^2} \times \sqrt{3}$ (from assumption) $\sqrt{27} \le \sqrt{a^2+b^2+c^2}$ $a^2+b^2+c^2 \ge 27$ as required.	**2**	2Mk: Provides correct proof. 1Mk: Identifies $\underset{\sim}{m} \cdot \underset{\sim}{n} \le \lvert\underset{\sim}{m}\rvert\lvert\underset{\sim}{n}\rvert$ or obtains $a^2+b^2+c^2 \ge ab+bc+ca$ by algebraic method.
16(c)	$\text{LHS} = \left(\frac{a}{b+c}+1\right)+\left(\frac{b}{a+c}+1\right)+\left(\frac{c}{a+b}+1\right)-3$ $= \frac{a+b+c}{b+c}+\frac{a+b+c}{a+c}+\frac{a+b+c}{a+b}-3$ $= (a+b+c)\left(\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a+b}\right)-3$ $= \frac{1}{2}\times((b+c)+(a+c)+(a+b))\times\left(\frac{1}{b+c}+\frac{1}{a+c}+\frac{1}{a+b}\right)-3$ $\ge \frac{1}{2}\times(3\times\sqrt[3]{(b+c)(a+c)(a+b)})\times\left(3\times\sqrt[3]{\frac{1}{(b+c)(a+c)(a+b)}}\right)-3$ (by AM/GM with $n=3$) $= \frac{9}{2}\times\sqrt[3]{1}-3$ $= \frac{3}{2}$ as required.	**3**	3Mk: Provides correct solution. 2Mk: Shows significant progress. 1Mk: Makes a valid attempt.
16(d)(i)	$y = x^k + (c-x)^k$ $\frac{dy}{dx} = kx^{k-1} - k(c-x)^{k-1}$ stationary points occur when $\frac{dy}{dx} = 0$ $kx^{k-1} - k(c-x)^{k-1} = 0$ $x^{k-1} = (c-x)^{k-1}$ $\left(\frac{c-x}{x}\right)^{k-1} = 1$ $\frac{c-x}{x} = \pm 1$ (note: if $k-1$ is even it is possible that $c-x$ is < 0) $\frac{c-x}{x} = -1$ or $\frac{c-x}{x} = 1$ $\frac{c}{x} - 1 = -1$ $\frac{c}{x} - 1 = 1$ $\frac{c}{x} = 0$ $\frac{c}{x} = 2$ not possible as $c > 0$ $x = \frac{c}{2}$ $\therefore y$ has a single stationary point at $x = \frac{c}{2}$	**2**	2Mk: Provides correct solution by considering both cases. 1Mk: Finds the stationary value of $x = \frac{c}{2}$.

16(d)(ii)	$\frac{d^2y}{dx^2} = k(k-1)x^{k-2} + k(k-1)(c-x)^{k-2}$ when $x = \frac{c}{2}$; $\frac{d^2y}{dx^2} = k(k-1)\left(\frac{c}{2}\right)^{k-2} + k(k-1)\left(\frac{c}{2}\right)^{k-2}$ $= 2k(k-1)\left(\frac{c}{2}\right)^{k-2}$ If $0 < k < 1$ then $k(k-1) < 0$ and if $k > 1$ then $k(k-1) > 0$ Thus if $k < 1$, y has a maximum at $x = \frac{c}{2}$ and if $k > 1$, y has a minimum at $x = \frac{c}{2}$.	**1**	1Mk: Provides correct solution.
16(d)(iii)	when $x = \frac{c}{2}$; $y = \left(\frac{c}{2}\right)^k + \left(\frac{c}{2}\right)^k$ $= 2\left(\frac{c}{2}\right)^k$ when $x \neq \frac{c}{2}$; for $0 < k < 1$; $x^k + (c-x)^k < 2\left(\frac{c}{2}\right)^k$ for $k > 1$; $x^k + (c-x)^k > 2\left(\frac{c}{2}\right)^k$ Let $c = a + b$ where $a > 0$, $b > 0$ and $a \neq b$ Thus $0 < a < c$ Case 1: $0 < k < 1$; $a^k + (c-a)^k < 2\left(\frac{c}{2}\right)^k$ $a^k + (a+b-a)^k < 2\left(\frac{a+b}{2}\right)^k$ $a^k + b^k < 2\left(\frac{a+b}{2}\right)^k$ $\frac{a^k + b^k}{2} < \left(\frac{a+b}{2}\right)^k$ Case 2: If k > 1; $x^k + (c-x)^k > 2\left(\frac{c}{2}\right)^k$ $a^k + b^k > 2\left(\frac{a+b}{2}\right)^k$ $\frac{a^k + b^k}{2} > \left(\frac{a+b}{2}\right)^k$	**3**	3Mk: Provides correct solution. 2Mk: Uses part (ii) to show that either $x^k + (c-x)^k > 2\left(\frac{c}{2}\right)^k$ or $x^k + (c-x)^k < 2\left(\frac{c}{2}\right)^k$ depending on the value of k. 1Mk: Finds the value of y when $x = \frac{c}{2}$.
	End of Paper		